education

Department:
Education
PROVINCE OF KWAZULU-NATAL

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2
PREPARATORY EXAMINATION
SEPTEMBER 2020
MARKING GUIDLINES

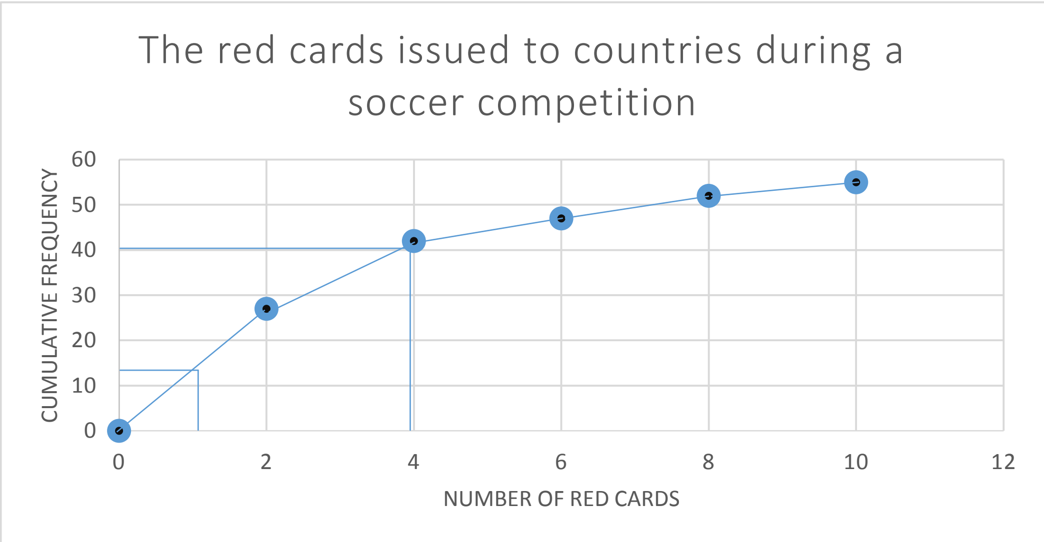
MARKS: 150

TIME: 3 hours

This marking guideline consists of 12 pages.

QUESTION 1

NUMBER OF RED CARDS	NUMBER OF COUNTRIES (f)	MIDPOINT OF INTERVAL (x)	$f.x$
$0 < x \leq 2$	27	1	27
$2 < x \leq 4$	15	3	45
$4 < x \leq 6$	5	5	25
$6 < x \leq 8$	5	7	35
$8 < x \leq 10$	3	9	27
TOTAL	55		159

1.1	Estimated mean $= \frac{159}{55} = 2,89 \approx 3$ red cards Answer only full marks	CA✓ 159 CA ✓ 55 CA ✓ answer (3)
1.2		✓✓✓Full marks for 6 correct points ✓✓2 marks for 4correct points ✓1 mark for 2 correct points (3)
1.3	$Q_3 = 4$ and $Q_1 = 1$ $\therefore IQR = 4 - 1 = 3$ red cards Answer only full marks	CA ✓ Q_1 and Q_3 CA✓ answer (2)
		[8]

QUESTION 2

2.1	$A = 5{,}97$; $B = 2{,}18$ $Y = 5{,}97 + 2{,}18\,\boldsymbol{x}$ Answer only full marks	A ✓ for A A ✓for B A✓✓ For equation (4)
2.2	Estimated monthly income $y = 5{,}97 + 2{,}18\,(9)$ $= 25{,}59$ ∴ Monthly income = R25598,89 If 9000 is used only 1 mark	CA✓ substitution CA✓ answer (2)
2.3	$r =$ 0,94	CA✓✓ (2)
2.4	Very strong positive relationship between the monthly rent and the monthly income.	CA ✓ strong CA ✓ positive (2)
		[10]

QUESTION 3

3.1.1	$m_{LM} = \frac{0-1}{4-1} = -\frac{1}{3}$ $m_{MN} = \frac{2-0}{8-4} = \frac{1}{2}$	A✓ sub into correct formula A ✓ $-\frac{1}{3}$ A✓ Sub into correct formula A ✓ $\frac{1}{2}$ (4)
3.1.2	$KM = \sqrt{(4-4)^2 + (10-0)^2}$ $= \sqrt{100}$ $= 10$ units Answer only full marks	CA ✓subst CA ✓10 units (2)
3.1.3	$m_{MN} = \frac{1}{2}$ $\tan\theta = \frac{1}{2}$ $\theta = 26{,}57°$ Answer only full marks	CA ✓$\tan\theta = \frac{1}{2}$ CA ✓ $\theta = 26{,}57^0$ provided acute angle (2)
3.1.4	$\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$ $\left(\frac{1+8}{2} ; \frac{1+2}{2}\right)$ $\left(\frac{9}{2} ; \frac{3}{2}\right)$	A✓correct substitution A✓answer (2)
3.2	$m_{KL} = \frac{10-1}{4-1} = 3$ $m_{KL} \times m_{LM} = 3 \times \left(-\frac{1}{3}\right)$ $= -1$ $\therefore KL \perp LM$	A✓subst A✓3 A✓product = −1 (3)
3.3	$m_{KN} = \frac{10-2}{4-8}$ $= -2$ $\therefore KN \perp NM$ $\therefore K\hat{L}M + K\hat{N}M = 180°$ $\therefore KLMN$ is cyclic quadrilateral (converse, opp $\angle^s$ of a cyclic quad are supplementary)	A✓M_{KN} -2 A✓KN ⊥ MN A✓Sum of 180° $M_{MN} = \frac{1}{2} \therefore (-2)\left(\frac{1}{2}\right) = -1$ A✓ reason (4) **[17]**

QUESTION 4

4.1	$M\left(\frac{-5+3}{2}\,;\,\frac{4+2}{2}\right) = M(-1;3)$	A✓ $x = -1$ A✓ $y = 3$ (2)
4.2	$r^2 = BM^2 = (-5+1)^2 + (4-3)^2 = 17$ $\therefore (x+1)^2 + (y-3)^2 = 17$	CA✓ subst into equation CA✓ $r^2 = 17$ CA✓equation For CA marks coordinates of M must be in second quadrant (3)
4.3	$m_{AB} = \frac{2-3}{3+1} = -\frac{1}{4}$ $m_{AN} = \frac{2+2}{3-2} = 4$ $m_{AB} \times m_{AN} = -1$ $\therefore B\hat{A}T = 90°$ $\therefore TA$ is a tangent (conv. tangent and diameter)	A✓m_{MA} or m_{BA} A✓m_{AN} A✓product of gradients = -1 A✓90^0 A✓reason (5)
4.4.1	$m_{TA} = m_{AN} = 4$ $y = 4x + c$ Subst. (3; 2): $2 = 4(3) + c$ $-10 = c$ $\therefore y = 4x - 10$	CA✓$m_{TA} = m_{AN}$ CA✓equation CA✓subst of (3; 2) or (2; -2) CA✓equation (4)
4.4.2	Let C(x; y) $\therefore (x+1)^2 + (y-3)^2 = 17$ At C; $x = 0$ $\therefore (0+1)^2 + (y-3)^2 = 17$ $(y-3)^2 = 16$ $y - 3 = \pm 4$ $y = 7$ or $y = -1$ $\therefore C(0; -1)$ $m_{BC} = \frac{-1-4}{0+5} = -1$ Now $y = -x - 1$	CA✓equation of circle CA✓subst x = 0 CA✓y values CA✓co-ordinate CA✓gradient CA✓equation (6)
4.5	Lines AT and BT intersect at C $\therefore 4x - 10 = -x - 1$ $5x = 9$ $x = \frac{9}{5} = a$ $b = -\frac{9}{5} - 1 = -2\frac{4}{5}$	CA✓equations equal CA✓value of a CA✓value of b, For CA marks A and B are points in the 4th quadrant (3)
		[23]

QUESTION 5

5.1	$\cos 79° \cos 311° + \sin 101° \sin 49°$ $= \cos 79° \cos 49° + \sin 79° \sin 49°$ $= \cos(79° - 49°)$ $= \cos 30°$ $= \frac{\sqrt{3}}{2}$ Answer only no marks, used calculator	A✓$\cos 49^0$ A ✓$\sin 79^0$ A✓$\cos 30^0$ A✓ answer (4)
5.2	$\sin(x+y) = 3\sin(x-y)$ $\sin x \cos y + \cos x \sin y$ $= 3(\sin x \cos y - \cos x \sin y)$ $\sin x \cos y + \cos x \sin y$ $= 3\sin x \cos y - 3\cos x \sin y$ $-2\sin x \cos y = -4\cos x \sin y$ $\div -2\cos x \cos y$: $\frac{\sin x}{\cos x} = 2\left(\frac{\sin y}{\cos y}\right)$ $\therefore \tan x = 2\tan y$	A✓expansion A✓like terms added A✓divide A✓ $\frac{\sin x}{\cos x} = 2\left(\frac{\sin y}{\cos y}\right)$ (4)
5.3.1	$\frac{\cos x}{\sin 2x} - \frac{\cos 2x}{2\sin x} = \sin x$ LHS: $\frac{\cos x}{\sin 2x} - \frac{\cos 2x}{2\sin x}$ $= \frac{\cos x}{2\sin x \cos x} - \frac{1-2sin^2x}{2\sin x}$ $= \frac{1}{2\sin x} - \frac{(1-2sin^2x)}{2\sin x}$ $= \frac{1-1+2sin^2x}{2\sin x}$ $= \frac{2sin^2x}{2\sin x}$ $= \sin x$ =RHS	A✓$2\sin x \cos x$ A✓$1-2sin^2x$ A✓numerator A✓answer (4)

5.3.2	$1+2\cos 2x = \dfrac{\cos 2x}{2\sin x} - \dfrac{\cos x}{\sin 2x}$ $1+2\cos 2x = -\sin x$ $1+2(1-2sin^2x) = -\sin x$ $1+2-4sin^2x = -\sin x$ $4sin^2x - \sin x - 3 = 0$ $(\sin x - 1)(4\sin x + 3) = 0$ $\sin x = 1$ OR $\sin x = -\frac{3}{4}$ $x = 90°$ $\text{ref}\angle = 48{,}59°$ $x = 228.59$ OR $x = 311{,}41°$	A✓$-\sin x$ A✓standard quadratic form A ✓ Factors CA✓90^0 CA✓228.59° CA✓311.41° (6)

QUESTION 6

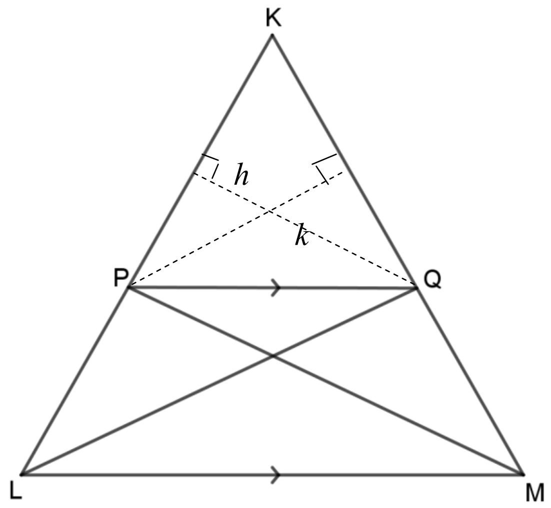
6.1	$a = 1$ $b = 2$ $c = 2$ $d = 1$	A✓$a = 1$ A✓$b = 2$ A✓$c = 2$ A✓$d = 1$ (4)
6.2	360°	A✓360° (1)
6.3.1	$x \in [-90°;\ 90°]$ *or* $x \in [270°;\ 360°]$	AA✓✓values and notation (2)
6.3.2	$x \in (-45°;\ 0°)$ *or* $x \in (45°;\ 90°)$ *or* $x \in (315°; 360°)$	AAA✓✓✓ values and correct notation (3)
		[11]

QUESTION 7

7.1	n ΔPQR: $\hat{Q}_1 = x$ $(PR = QR)$ $\hat{R} = 180° - 2x$ $(sum\ of\ \angle\ \Delta PQR)$ $Area\ of\ \Delta PQR = \frac{1}{2}pq\sin\hat{R}$ $= \frac{1}{2}m.m\sin(180° - 2x)$ $= \frac{1}{2}m^2\sin 2x$	A✓$\widehat{Q_1} = x$ A✓$\widehat{R} = 180° - 2x$ A✓Subst. into Area rule A✓sin2x A✓answer (5)

7.2	$\therefore \frac{PQ}{\sin(180° - 2x)} = \frac{m}{\sin x}$ $\therefore PQ = \frac{m\,.\sin(180° - 2x)}{\sin x}$ $\therefore PQ = \frac{m\,.\sin 2x}{\sin x}$ $\therefore PQ = \frac{m\,.2\sin x\,.\cos x}{\sin x}$ $\therefore PQ = 2m\cos x$	A✓Use of sine rule A✓subst into sine Rule A✓sin $2x$ A✓2 sin x cos x (4)
7.3	In ΔSPQ: $\tan y = \frac{SP}{PQ}$ $\therefore SP = PQ\tan y$ $\therefore SP = 2m\cos x\tan y$	A✓$\tan y = \frac{SP}{PQ}$ A✓SP = PQ tan y (2)

QUESTION 8
8.1



R.T.P	$\frac{KP}{PL} = \frac{KQ}{QM}$	
	CONSTRUCTION: In ΔKPQ, draw perpendicular heights, h from Q to KP and K from P to KQ	A✓construction
	$\frac{\text{Area of } \Delta KPQ}{\text{Area of } \Delta LPQ} = \frac{\frac{1}{2}KP \times h}{\frac{1}{2}PL \times h}$	A✓method
	$= \frac{KP}{PL}$	A✓ $\frac{KP}{PL}$
	$\frac{\text{Area of } \Delta KPQ}{\text{Area of } \Delta MQP} = \frac{\frac{1}{2}KQ \times k}{\frac{1}{2}QM \times k}$	A✓method
	$= \frac{KQ}{QM}$	A✓ $\frac{KQ}{QM}$
	But area of ΔPLQ = Area of ΔMPQ Same base, same height	
	$\therefore \frac{\text{Area of } \Delta KPQ}{\text{Area of } \Delta LPQ} = \frac{\text{Area of } \Delta KPQ}{\text{Area of } \Delta MQP}$	A✓method
	$\therefore \frac{KP}{PL} = \frac{KQ}{QM}$	(6)

8.2.1	In ΔAPQ: BC ∥ PQ $\frac{AB}{AP}=\frac{AC}{AQ}$.; conv prop $\hat{T}_1=\hat{C}_2$ alternate ∠s; BC ∥ PQ $\hat{A}_2=\hat{C}_2$ tangent TC; chord BC $\therefore \hat{A}_2=\hat{T}_1$	A✓S A✓R A✓ S/R A✓ S/R (4)
8.2.2	In ΔABC and ΔTCQ: $\hat{C}_3=\hat{Q}$ corr∠ˢ; BC ∥ PQ $\hat{A}_2=\hat{T}_1$ proved above $\hat{B}_2=\hat{C}_1$ rem ∠ˢ ∴ ΔABC|||ΔTCQ ∠∠∠	A✓ S/R A✓ S/R A✓ S/R A✓ S/R (4)
8.2.3	$\hat{B}_1=\hat{C}_3$ tangent SB; chord AB $\hat{Q}=\hat{C}_3$ proven $\therefore \hat{B}_1=\hat{Q}$ ∴ ABTQ is cyclic conv. ext ∠ = int∠of cyclic quad.	A✓S A✓R A✓ S A✓ S/R (4)
8.2.4	TB = TC tangents from common point $\hat{B}_3=\hat{C}_2$ TB = TC; ∠s opp eq. sides $\hat{T}_1=\hat{C}_2$ alt. ∠s; BC ∥ PQ $\therefore \hat{B}_3=\hat{T}_1$ ∴ TQ is a tangent conv. tan; chord theorem	A✓S A✓R A✓S A✓S/R A✓S/R (5)
		[23]

QUESTION 9

9.1	In ΔMBC: $\hat{B}_2=\hat{B}_3=x$ BE bisects $M\hat{B}C$ $\therefore M\hat{B}C=2x$ $M\hat{B}C=M\hat{C}B=2x$ angles opposite equal sides In ΔBEC: $\hat{E}_2=180^\circ-(x+x)$ Sum of angles of a Δ $=180^\circ-2x$	 A✓S A✓S/R A✓S/R A✓Answer (4)

9.2	In $\Delta MBC: B\hat{M}C = 180^\circ - (2x+2x)$ Sum of angles of a Δ $= 180^\circ - 4x$ But $B\hat{A}C = \frac{1}{2} B\hat{M}C$ $\angle$ at centre twice angle $= \frac{1}{2}(180^\circ - 4x)$ $= 90 - 2x$	A✓S A✓R A✓S/R (3)
9.3	In ΔABE: $\hat{E}_1 + \hat{E}_2 = 180^\circ$ Straight line $\hat{E}_1 = 180^\circ - E_2$ $= 180^\circ - (180^\circ - 2x)$ $= 2x$ In ΔABE: $A\hat{B}E + B\hat{A}C + \hat{E} = 180^\circ$ Sum of $\angle$s of Δ $A\hat{B}E = 180^\circ - (B\hat{A}C + \hat{E}_1)$ $= 180^\circ - (90^\circ - 2x + 2x)$ $= 90^\circ$ $\therefore$ AE is a diameter of circle ABE (Subtends) $\angle$ 90°	A✓S/R A✓S A✓S/R A✓S A✓R (5)
		[12]

QUESTION 10

10.1.1	Let $\hat{Y}_1 = a$ and $\hat{N} = b$ $\therefore \hat{T}_3 = a - b$ (ext. ∠ of Δ = sum opp. ∠s) $\hat{T}_1 = \hat{N} = b$ (tan XT; chord MT) $X\hat{T}Y = a$ (angles opposite equal sides) $\hat{T}_2 = X\hat{T}Y - \hat{T}_1$ $= a - b$ $\therefore \hat{T}_3 = \hat{T}_2$ $\therefore$ YT bisects $M\hat{T}N$	A✓ S/R A✓S A✓R A✓ S/R A✓S (5)
10.1.2	In ΔXMT and ΔXTN: $\hat{X}$ is common $\hat{T}_1 = \hat{N}$ tan XT; chord MT $\hat{M}_1 = X\hat{T}N$ remaining ∠ $\therefore \Delta XMT \parallel\!\mid \Delta XTN$ ∠∠∠ $\therefore \frac{XM}{XT} = \frac{XT}{XN} = \frac{MT}{TN}$ similar Δ's $\therefore \frac{XM}{XT} = \frac{XT}{XN}$	A✓S/R A✓S A✓R A✓R A✓R A✓ S/R (6)
10.2.1	$XM = XY - 20$ XY = XT $= k - 20$	A✓S A✓R A✓answer (3)
10.2.2	$\frac{XM}{XT} = \frac{XT}{XN}$ $\therefore \frac{k-20}{k} = \frac{k}{k+50}$ $\therefore (k-20)(k+50) = k^2$ $\therefore k^2 + 30k - 1000 = k^2$ $\therefore 30k - 1000 = 0$ $\therefore 30k = 1000$ $\therefore k = 33{,}3$ mm	A✓ LHS A✓ RHS A✓Simplification A✓Answer (4) **[18]**

TOTAL: 150